Dynamic Gear Loads due to Coupled Lateral, Torsional and Axial Vibrations in a Helical Geared System.

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ABSTRACT

In this paper we investigate the rotordynamics of a geared system with coupled lateral, torsional and axial vibrations, with a view toward understanding the severe vibration problems that occurred on a 28-MW turboset consisting of steam turbine, double helical gear and generator. The new dynamic model of the shaft line was based on the most accurate simulation of the static shaft lines, which are influenced by variable steam forces and load-dependent gear forces. The gear forces determine the static shaft position in the bearing shell. Each speed and bad condition results in a new static bending line which defines the boundary condition for the dynamic vibration calculation of the coupled lateral, torsional and axial systems. Rigid disk and distributed springs were used for shaft line modeling. The tooth contact was modeled by distributed springs acting normally on the flank surfaces of both helices. A finite element method with distributed mass was used for lateral and torsional vibrations. It was coupled to a lumped mass model describing the axial vibrations. The forced vibrations due to unbalances and static transmission errors were calculated. The eigenvalue problem was solved by means of a stability analysis showing the special behaviors of the coupled system examined. The calculation was successfully applied, and the source of the vibration problem could be located as being a gear-related transmission error. Several redesign proposals lead to a reliable and satisfactory vibrational behavior of the turboset.

1 Introduction

Some unexpected vibration phenomena occurring on a geared turboset could not be explained completely by conventional methods. The shaft outline of a low- and high-speed string is shown in Fig.1. Only one axial bearing at the hot end of the 28-MW steam turbine defines the position of the gear and generator in axial direction, while the couplings on the high-speed and low-speed sides are of the normal flange design. The gear mesh of the double helical gear accommodates the low axial forces which may result from axial misalignment of the generator rotor in the magnetic field of the stator. Good operational experience gathered so far from more than 100 power plants of the same string concept have proved that this design is reliable and easy to install as long as conventional design rules, by which resonance conditions are avoided, are adhered to.

With arrangements of this kind, the vibrations may be excited by normal unbalance forces of each rotor as well as by a geometrical misalignment of the flange couplings. It is a well known fact that all unbalance response calculations and stability evaluations of such shaft line configurations are influenced by the loading of the gear mesh as the bearing stiffness increases with the growth of the load. This behavior will change the resonance tuning of the highspeed shaft line. Therefore it is necessary to consider static effects in the described arrangement at all speed and load conditions.

Torsional frequencies may be excited by oscillating generator moments with short-circuit or electrical fault conditions. Vibrations of the transmission angle due to small deviations of the accumulated pitch error of the gear mesh can also excite the system with all harmonic portions inherent in the spectrum of the pitch errors of the pinion and the bull wheel. Such deviations may excite lateral, torsional and axial vibrations.



Fig. 1 Shaft and bearing arrangement of a 28 MW turboset (ABB Turbinen Nürnberg, Germany)

Current practice is to handle the vibration phenomena of both shaft lines separately by 2-dimensional rotordynamics (Mittwollen et al., 1991 / Han et al., 1995), considering the special alignment situation and the additional bearing forces caused by the moment transmission in both wheels. The bearing data are depending on the load and the load angle and have to be carefully calculated for each speed and load case with external forces acting on the bending line. This procedure is described by Hubensteiner et al. (1994) using proven codes for bearing data (Han, 1983 / Mittwollen, 1991) in a 2-dimensional application with interpolation for load and load angle. Carefully examining the force transmission and the real possibility of motion of the gearconnected shafts, it is obvious that both shafts can only vibrate in a combined motion, as the gear mesh behaves much more stiffly than the rotor journal bearings.

In the case of the example shown in Fig. 1, two phenomena occurred which could not be α plained by the normal rotor dynamics of the uncoupled system. The first was a coupled torsional and bending instability at a low 10% load, in which the shaft vibrated with the first torsional frequency. This frequency was obtained from the eigenvalue analysis of torsional vibrations. This phenomena could be explained by the possible occurrence of a normally unexcitable horizontal lateral frequency of the shaft which is supported in 2-lobe bearings. Due to the gear kinematics, the shaft is forced to vibrate in a nearly horizontal mode with lower damping.

The more severe vibrations occurred in axial direction with very high 50-Hz amplitudes at the axial bearing of the turbine, which was running at 98 Hz nominal speed. These vibrations were transmitted from the generator and wheel set, though their vibration values amounted to only half of the axial turbine vibration amplitudes. The main goal of this investigation is to explain this behavior with the help of a complete static and dynamic model excited by unbalances and gear transmission errors. The transmission error is defined, for any instantaneous position of one gear, as the position departure of the mating gear from the position it would occupy if the gear teeth were rigid and the system were perfect. (Kishor and Gupta , 1989)

2 State of the art in multi-string rotor dynamics

Coupled vibrations and instabilities were found in a number of quite different geared machine by Yamaha, Mist (1979), Rachel and Szenasi (1980). The theory of coupled vibrations due to gear pair has been studied by many researchers. Iannuzzelli and Elward (1984) describe in their study that some measured eigenfrequencies of a geared compressor can be verified only by considering the torsional and lateral coupling due to the gear. Simmon and Smalley (1984) found by experimental and analytical investigations that some unexpected high torsional damping values are associated with high lateral vibrations. Iida et al (1980) base their study on a simple geared-rotor system including the effects of torsional-flexural coupling. This study only considers the flexibility of the driven shaft acting together with a stiff driving shaft. Neriya et al (1984) extended this study by considering the flexibility of the driving shaft as well. Schwibinger et al (1988) concluded that the coupling not only affects the eigenfrequencies and modes, but also the forced vibrations due to mass unbalances in their study. Kishor and Gupta (1989) examined the dynamic gear load in a simple geared rotor supported by hydrodynamic journal bearings. In this study the effects of mass unbalance and transmission errors were calculated, including the contact loss problems, by using the time integration method. Choy et al (1991) extended their previous study by including the effects of gear box motions. They used modal analysis to reduce the degrees of freedom, and applied the time integration method. The influence of gear forces on the static behavior of a geared shaft line was studied by Hubensteiner et al. (1994), where questions of the shaft arrangements on static and dynamic behavior were investigated. The new modeling which includes the axial vibrations is derived for a simple helical geared system by Blankenship et al. (1995) and Kahraman (1994).

However, only the coupled effect between lateral and torsional vibrations or only a simple mass model with axial included vibrations was considered in most of the earlier studies. Therefore, new calculation models that include the coupled axial vibrations were developed for a helical gear to simulate and explain the measured vibrations. The main goal was to find out the realistic overload factor of the gear mesh due to this special shaft arrangement.

The gear pairs were modeled by rigid disks and distributed springs along the contact line, and the shafts were modeled using the finite element method and the lumped mass method. The amplitudes of vibrations and dynamic forces due to mass unbalances and transmission errors were calculated and compared with the measurement. The damped eigenvalue problems were solved by means of a stability analysis to explain the actually observed behavior.

3 Analytical modeling of the coupled system 3.1 Static shaft lines

The mass and stiffness distribution of the rotor system defines the bending lines of the multi bearing shaft arrangement in a conventional way. But the actual situation has to be simulated carefully based on three steps: search of the basic bearing position, equilibrium of the bending line support forces in the bearing shell and finally calculation of the bending line influenced by the oil film.

The static bearing position of each bearing shell in a multibearing line is defined by the ideal connection of the coupling flanges without angle deviation and free of eccentricity. This is demonstrated in <u>Fig. 2</u>. In horizontal direction the bearings are to be mounted in an ideal strait line. Deviations due to heat expansion or other effects have to be evaluated separately and finally taken into account well defined shift of the bearings away from the here defined basic values. Starting with the bearing loads calculated from the above mounting position the rotor equilibrium on the bearing oil film can be found for a defined speed and external load acting on the shaft.



(a) view from side



(b) view from top Fig. 2 Ideal alignment of shaft lines

The change of the equilibrium point in the journal bearings with load makes it necessary to recalculate the shaft bending lines, producing a new result as input for the above shaft position in the bearing shell. This example demonstrates that an iteration has to be applied to find the actual static positions of the coupled shaft lines taking into account the bearing properties at each speed and load point. The latter depends on the gear force, and also on the load-dependent forces acting in the control stage of the steam turbine. At each equilibrium of the shaft line the linear stiffness and damping coefficients of the journal bearing are defined, and can finally be used in linear rotordynamics calculating forced response and eigenvalues of the gear coupled rotors.

3.2 Vibration model of the coupled shaft line

Finite element modeling is used to analyze the lateral and torsional vibrations, considering the gyro and shear effects (Han et al., 1995). The finite element consists of two nodes, where each node has 5 degrees of freedom by way of two lateral displacements, two bending rotation angles, and a torsional rotation angle. A lumped mass modeling is used for the analysis of axial vibrations. Then the equations of motion of a typical rotor bearing system can be written as,

$$[M]\{\mathfrak{P} + \left([C] - \sum_{i} \Omega_{i}[G_{i}] \right) \{\mathfrak{P} + [K]\{q\} = \{f\}$$

$$\tag{1}$$

where the global displacement vector $\{q\}$ and the force vector $\{f\}$, based on the nodal coordinates system as shown in <u>Fig. 3</u>, are represented by

$$\left\{q\right\} = \begin{cases} \left\{q_a\right\}\\ \left\{q_b\right\}\\ \left\{q_t\right\} \end{cases}, \quad \left\{f\right\} = \begin{cases} \left\{f_a\right\}\\ \left\{f_b\right\}\\ \left\{f_b\right\}\\ \left\{f_t\right\} \end{cases}$$

$$(2)$$



Fig. 3 Nodal coordinate system of a finite shaft element

3.3 Gear modeling

The gear pair is modeled by rigid disks and linearly distributed springs along the contact lines. The gear pair used for analytical modeling is shown in Fig. 4, where α and β are transverse pressure angle and helical angle respectively. The displacement vector of the contact points can be written as a function of s, which is a local coordinate to the axial direction.

$$\widetilde{\boldsymbol{d}}(s) = (x_i + r_i \boldsymbol{q}_j) \widetilde{\boldsymbol{e}}_x + (y_i + s \boldsymbol{q}_j) \widetilde{\boldsymbol{e}}_y + (z_i - r_i \boldsymbol{q} - s \boldsymbol{q}_{ji}) \widetilde{\boldsymbol{e}}_z$$

$$\widetilde{\boldsymbol{d}}_j(s) = (x_j - r_j \boldsymbol{q}_j) \widetilde{\boldsymbol{e}}_x + (y_j + s \boldsymbol{q}_j) \widetilde{\boldsymbol{e}}_y + (z_j + r_j \boldsymbol{q}_j - s \boldsymbol{q}_{jj}) \widetilde{\boldsymbol{e}}_z$$
(3)

Then the normal displacements of the contact points can be obtained by using of following vector calculations.

$$\overset{\omega}{\boldsymbol{d}}_{n} = \left(\overset{\omega}{\boldsymbol{d}} \cdot \vec{\boldsymbol{e}}_{n} \right) \vec{\boldsymbol{e}}_{n}, \quad \overset{\omega}{\boldsymbol{d}}_{jn} = \left(\overset{\omega}{\boldsymbol{d}}_{j} \cdot \vec{\boldsymbol{e}}_{n} \right) \vec{\boldsymbol{e}}_{n}$$
(4)

where, $\hat{\mathbf{e}}_n$ is a unit normal vector of the contact point defined as $\vec{\mathbf{e}}_n = \sin \mathbf{b} \vec{\mathbf{e}}_x + \sin \mathbf{a} \cos \mathbf{b} \vec{\mathbf{e}}_y + \cos \mathbf{a} \cos \mathbf{b} \vec{\mathbf{e}}_z$ (5)



Fig. 4 Schematic diagram of a gear pair

Using equation (3), (4) and (5) we can get \hat{e} the normal displacement vectors as follows.

$$\mathbf{a}_{in}^{(s)} = \left((x_i + r_i \mathbf{q}_{ji}) \sin \mathbf{D} + (y_i + s \mathbf{q}_{ji}) \sin \mathbf{a} \cos \mathbf{D} + (z_i - r_i \mathbf{q} - s \mathbf{q}_{yi}) \cos \mathbf{a} \cos \mathbf{D} \right) \vec{e}_n$$

$$\overset{\text{tot}}{\mathbf{d}_{jn}} (s) = \left((x_j - r_j \mathbf{q}_{jj}) \sin \mathbf{D} + (y_j + s \mathbf{q}_{jj}) \sin \mathbf{a} \cos \mathbf{D} + (z_j + r_j \mathbf{q}_j - s \mathbf{q}_{yj}) \cos \mathbf{a} \cos \mathbf{D} \right) \vec{e}_n$$

$$+ (z_j + r_j \mathbf{q}_j - s \mathbf{q}_{yj}) \cos \mathbf{a} \cos \mathbf{D}) \vec{e}_n$$
(6)

Then the normal reaction forces for the infinitesimal element can be as given below

$$dF_n = k(s) \left| \vec{\mathbf{d}}_{n}(s) - \vec{\mathbf{d}}_{n}(s) \right| ds$$
⁽⁷⁾

where k(s) is the specific stiffness. Now, we consider linear distributions of the mesh stiffness for simplifying the modeling as shown in Fig. 5. Then any nonlinear distribution can be solved by superposition of the linear distribution results. The functions of the distributions are expressed as

$$k(s) = \frac{k_b - k_a}{b - a}(s - a) + k_a$$
(8)

As we can see from equation (7), the total reacting forces due to displacement can be derived using integration as follows.

$$\begin{array}{l} \overset{\omega}{F_{in}} = \int dF_n \vec{e}_n = -\vec{F}_{jn} \\ \overset{\omega}{T_{in}} = \int dF_n (\overset{\omega}{r_i} \times \vec{e}_n) \\ T_{jn}^{\vec{\omega}} = -\int dF_n (\overset{\omega}{r_j} \times \vec{e}_n) \end{array} \tag{9}$$

The basic stiffness matrix can be obtained by arranging the above results in the following equation.

$$\begin{bmatrix} K_G^B \\ x_i x_j y_i z_i \mathbf{q}_{ii} \mathbf{q}_{ij} y_j z_j \mathbf{q}_{ij} \mathbf{q}_{ij} \mathbf{q}_{ij} \mathbf{q}_{ij} \end{bmatrix}^T = \{ f_G \}$$
(10)

<u>Fig. 5</u> Assumed linear distribution of the stiffness along contact width

Now we consider a coordinate transformation of a rotation γ which is given in Fig. 6. Then the general stiffness matrix [*K*_{*C*}] of the gear pair can be written as

$$\begin{bmatrix} K_G \end{bmatrix} = \begin{bmatrix} R(\mathbf{g}) \end{bmatrix}^T \begin{bmatrix} K_G^B \end{bmatrix} \begin{bmatrix} R(\mathbf{g}) \end{bmatrix}$$
(11)

where $\left[R(\mathbf{g})\right]$ is a general transformation matrix for the rotation γ .

Considering static transmission errors in the gear meshes, such as pitch errors or axial apex wobbles, which are the result of above errors, the contact points will vary as a function of rotating angle. So the reaction force of the mesh can be rewritten as follows (Kishor and Gupta, 1989).

$$[K_G](\{q\} - \{p_G\}) = \{f_G\}$$
(12)

where $\{q\}$ is the displacement vector of the two gears' nodes and $\{p_G\}$ is the vector of transmission error.

The expression for the transmission errors can be rewritten by using the Fourier transformation.

$$\{p_G\} = \sum_{i} \left\{ p_{Gci} \right\} \cos \Omega_i t + \{p_{Gsi}\} \sin \Omega_i t \right)$$
(13)

After combining all of the above equations, the following equation of motion results.

$$\begin{bmatrix} M \end{bmatrix} \left\{ \stackrel{\cdot}{q} \right\} + \left[\begin{bmatrix} C \end{bmatrix} - \sum_{i} \Omega_{i} \begin{bmatrix} G_{i} \end{bmatrix} \right] \left\{ \stackrel{\cdot}{q} \right\} + \left[\begin{bmatrix} K \end{bmatrix} + \begin{bmatrix} K_{G} \end{bmatrix} \right] \left\{ q \right\} = \left\{ f \right\} + \begin{bmatrix} K_{G} \end{bmatrix} \sum_{i} \left\{ \left\{ p_{Gci} \right\} \cos \Omega_{i} t + \left\{ p_{Gsi} \right\} \sin \Omega_{i} t \right\}$$
(14)



Fig. 6 Coordinate transformation of the location angle

4 Numerical Results

4.1 Unbalance calculation

The calculation model of the turboset has 37 nodes of lateral and torsional vibrations and 5 nodes for the axial directions. Thus, they have 200 degrees of freedom in all. The dynamic coefficients of the seals and bearings with respect to speed and load are calculated for all static equilibrium conditions which mean a steady state operating point for a given speed and loading condition. The data of unbalances are assumed as the maximum of the very unbalanced turbine rotor and generator rotor, which is recommended by the company. The actual values are given in Fig. <u>7</u>.

We first calculate the amplitude of vibrations and the dynamic gear force due to unbalances. The results of the unbalance response are shown in <u>Fig. 8</u> assuming a turbine unbalance and <u>Fig. 9</u> for an unbalance in the generator rotor. The abscissa was split into an interval of increasing speed from 0 to 1, meaning 100% operational speed, and into a load portion from 0% to 100% load, exactly as the turbine is actually operated. All calculation results are about the vibrations at the midspan of each component as shown in Fig. 7.

From the results of Fig. 8 we see that the coupled effects due to the turbine unbalances are just slightly visible by amplitudes of the rotors being away from the turbine. This means that the amplitudes due to a turbine unbalance are nearly zero in the generator. The dynamic gear forces will only amount to 0.4 percent of the static value in this case.

A high generator unbalance which is assumed in Fig. 9 may cause small lateral vibration amplitudes of the turbine. The dynamic gear forces reach a value of 1.2 percent of the static force only. We do not expect any gear failures from those unbalances.

The axial amplitudes excited by the generator unbalance will lead to a tilting motion of the wheel and thereby excite axial vibrations of the shaft line. We see that the axial amplitudes at the turbine are even higher than those at the pinion and the wheel. It should be mentioned that this is a forced motion and must not be caused by a special resonance condition. The effects of amplitudes being higher at the turbine then those of the wheel can be explained by the different size of masses connected by the shaft spring. The value of amplitudes, however, is with 5 μ m op far below the actual measured value of 35 μ m o-p.



Fig. 7 Calculation system with definition of unbalances and amplitude margins.







4.2 Excitation due to transmission errors

In order to find an explanation for the high axial vibrations at the turbine bearing, further investigations were made by applying transmission errors of the gear. Those errors are inherent in each gear due to the normal manufacturing tolerances. In the actual case two identical turbosets showed completely different behavior in their axial vibrations and also the different reliability resulting from them.

The reason for it could be derived from the difference of the measured accumulated pitch error. One turbo gear had 8.5 μm o-p amplitude of accumulated pitch error, while the other had only less than 4 μm o-p. The two machines showed completely different vibration and wear effects, which were problematic for one turboset but not for the other.

Now, the main issue of this investigation was to determine the height of the dynamic gear forces and axial vibrations due to transmission errors. The circumferential variation of the single pitch error of each tooth depends on the manufacturing procedure. We performed the calculations for the worst case of transmission errors, i.e., with 8.5 μm o-p pitch error and also resulting from this 10 µm o-p axial error. Both errors were estimated with an additional worst case factor proportional to the measurements of the tooth geometry taken from the usual quality control reports. The results of this calculation are shown in Fig. 10. The axial vibrations at the turbine with 38 µm o-p at full load and 20 µm at partial load are very similar to the measured values. The overload factor of the gear can be easily derived from this calculation, also shown in Fig. 10. As we see, the dynamic tooth forces of the gear may be up to 15 percent of mean static forces. On one hand, this is a rather high value describing the overload produced by the geometry of the wheel set which is stiffly fixed between two heavy masses. On the other hand, the value is small as it should be covered by the overload factor of the static gear design calculation. The reason for the load dependency of the vibrations described in the above figure can easily be explained by the increase of gear forces causing an increase of bearing forces and bearing stiffness. Therefore, the resonance frequency of the dominant axial mode gets closer to the operating speeds. Additional calculations proved that the above described pitch errors with higher frequencies also give cause for worry because, with lower failure amplitudes exciting higher frequencies, they can produce nearly the same overload factor. Also, resonance conditions can occur with these higher frequencies and cause much higher gear overloads. To avoid overload to the gear, the pinion or the gear wheel must be vibrationaly separated from the heavy masses of the turbine or the generator. This can be achieved by employing an axially very flexible diaphragm coupling instead of the rather stiff intermediate shaft at the location of C1 and C2. Also, the bearing types were changed to avoid the instabilities. The changes are described by offset-halves instead of the 2-lobe type bearings for the pinion, and 2-lobe instead of cylindrical bearings for the generator. Compared

with Fig. 10, the success of these measures can be clearly seen in Fig. 10. Here the pinion still vibrates with the same almost 20 μ m amplitude in axial direction, but the turbine vibrations disappear as well as the additional gear forces fall to a negligible value.

This excellent improvement has been achieved with the strategy of vibration isolation of the exciting motion from the heavy mass. Systematic investigations can also be very helpful in understanding the main influences on the vibration system. The variable springs are in this case that of the axial bearing and that of the coupling between turbine and pinion. Parameter studies have been made to check the influence of both springs in Fig 12 and Fig. 13. The no bad and the full load case have been taken as representative in this case. The basis value of axial bearing spring was 1000 kN/mm and the same value for the original design of the coupling being rather soft due to a chambered flange design having only contact at the flange outer diameter. The variation of the coupling stiffness shows in Fig. 12 the easy understandable behavior that a softer coupling can isolate the pinion vibrations from the turbine. Lower stiffness decreases the amplitudes of the turbine and as well the gear forces step by step. Varying the stiffness of the axial bearing at the turbine we see in Fig 13 some not so homogenous behavior. The turbine and pinion amplitudes increase with lower axial bearing stiffness. This means that the above in Fig. 10 reported influence of the gear transmission error can have nearly twice the above value due to a variation of the axial bearing stiffness. As it is in fact not exactly possible to calculate the stiffness of the axial bearing, the load of which is low due to a small and variable steam force, a certain application factor has to be considered too in the evaluation of damage risk of the original system. Nevertheless the main clause of separating the pinion vibrations from the turbine by applying a softer coupling is valid and effective for all axial bearing stiffness.

4.3 Stability behavior

The unstable behavior of the shaft line could not be detected and eliminated by the normal 2-dimensional rotor dynamics as described above. The new combined method shows in Fig. 14 the real and imaginary parts of the eigenvalues, denoted by the decay factor u and the frequency v respectively. Negative damping (-u/v < 0) means instability which is identified by the hatched area. It can be seen that the instable region disappears after nearly 8 percent load, a fact which is in accordance with the measurements.

In the redesigned configuration of the string, we changed the couplings, used 2-lobe bearings in the generator, and offset-halves bearings for the pinion in order to eliminate the instability. The calculation results for the new design are shown in <u>Fig. 15</u>. In both figures Fig. 14 and 15 only the approximately 19 Hz frequency producing the instability was selected. This frequency is identical with the first torsional frequency of the system. The instability marked by the hatched area was eliminated successfully.



Fig. 10 Vibrations due to transmission errors of the original version











<u>Fig. 14</u> The eigenvalue of the unstable \approx 19 Hz frequency in original configuration



Fig. 15 The eigenvalue of the \approx 19 Hz frequency in redesigned version

5 Conclusion

The new analytical model for a helical geared system was developed considering the coupling between lateral, torsional and axial directions. The turboset, on which heavy axial vibrations occurred, was investigated and also in the actual plant successfully redesigned by using the new 2-string modeling method. The high axial vibrations and the high gear forces due to transmission errors can be simulated and the results are very similar to the measured vibrations. The instability which could not be detected by conventional calculations was found as well by using the new calculation model.

The reason for the high axial vibrations of the geared 2-string rotordynamic system could not be explained by assuming normal unbalances in the calculations. It was only after the gear transmission failures had been taken into account that the real source of vibrations was detected. The design modifications are successfully accomplished by changing the coupling and bearings in order to reduce the dynamic forces of the gear and to eliminate the instability by applying more stable bearing types.

Only the calculation of the gear coupled shaft line can detect the real source of this vibration system. Conventional methods of twodimensional rotordynamics cannot describe the occurring gear forces which can not be caused by normal unbalances of the gear connected machines in the investigated case. But transmission errors of the gear can lead to high dynamic transmission forces. The value of those dynamic overload is relatively high but not in an absolute dangerous margin for the gear as long as resonance conditions will not occur.

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Nomenclature

[M], [C], [G], [K]	mass, damping, gyroscopic and stiffness matrix respectively.
$[K_G]$	stiffness matrix due to gear mesh
$\{q_a\}, \{q_b\}, \{q_t\}$	generalized displacement vector to the direction of axial, lateral and torsional respectively
$\{\hat{f}_a\}, \{\hat{f}_b\}, \{f_t\}$	generalized displacement vector to the directions of axial, lateral and torsional respectively
`d n	displacement vector of the contact point in the i-th stage to the normal direction
a b and g	pressure angle, helical angle and position angle of a gear
S	local coordinate of a gear
r_i	radius of the gear i
X, Y, Z	global coordinates
(x_i, y_i, z_i)	displacement vector of the i-th gear node to the direction of
$(\boldsymbol{Q}^{T}\boldsymbol{Q}_{i}, \boldsymbol{Q}_{i})$	rotational vectors of the i-th gear node
k(s)	specific stiffness of the mesh stiffness
\mathcal{F}_{in} , \mathcal{T}_{in}	vectors of normal forces and torsion of the i-th gear due to deflections.
\hat{e}_n	normal vector of the contact point in the i-thgear
W	rotational speed of the i-th rotor
$\{e_G\}$	the vector of static transmission errors in the gear
U	amount of unbalance